All students should… explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way."

"Changing Garden"
Grades 4-5

GOALS

- Calculate the perimeter of a rectangle
- Understand that for one given perimeter, many different areas are possible.

PRIOR KNOWLEDGE

Students should have had experience in measuring area with informal and nonstandard units and in exploring the characteristics of rectangles.

MATERIALS AND EQUIPMENT

- A copy of the blackline master "Changing Garden" for each student.
- Two or three pictures of gardens and garden fences from gardening magazines, books or catalogs.
- One sheet of one-inch grid paper for each student
- Twenty to thirty square-inch tiles for each pair of students

CLASSROOM ENVIRONMENT

The students work in pairs on the activity and then come together as a class to discuss their work.

ACTIVITY

Engage

Show the class different pictures of gardens, and let the students share stories about gardens and gardening. Ask questions such as the following:

- "What types of gardens have you planted?"
- "What are some uses of gardens?"
- "What tasks are involved in planting a garden?"

Tell your students to suppose that they are designing a rectangular flower and vegetable garden that they could plant at school or at someone's house. Ask, "Do you think we might want to put a fence around our garden?" If your students say yes, ask them why. (Garden fences keep out animals that eat plants.

Fences also prevent people and animals from running through gardens and accidentally stepping on plants.)

Pursue the subject: "What materials do people use to make garden fences?" After your students have made a few suggestions, tell them that people often use wire mesh to make fences around their gardens.

Explain that wire mesh is inexpensive and lets in sunlight, and some garden plants, like cucumbers, grow in vines that can climb on the wire. Ask the students to think about wire fencing for the garden that they are designing.

Explore

Tell the students to suppose that they have 30 feet of fencing to mark out and protect a rectangular garden. Remind the students that *perimeter* means the measurement around a figure, so the perimeter of this garden will be 30 feet. The students' task is to figure out the different rectangles that have a perimeter of 30 feet, given the following conditions:

- The fence must have no gaps or overlaps.
- The length of each side of the rectangular garden must have a whole-number measure.

Spend a few moments finding classroom objects with two-dimensional shapes that have noncurved sides. Try to include regular and irregular shapes as well as shapes that are concave and convex. Talk about the perimeters of these shapes.

Distribute the blackline master "Changing Garden." Group the students in pairs for the activity, and give each pair a sheet of one-inch grid paper and about thirty square tiles to use in finding all possible rectangles that fit the requirements. Say, "For this activity, let's agree that one inch on the grid paper and one inch on the side of a tile will represent one foot of fencing." Encourage the students to look for relationships between the perimeter (30 feet) and the length and width of each rectangle that satisfies the given conditions.

Before the pairs begin to work, review the difference between area and perimeter, as well as how to apply the formula for area of rectangle. Ask the students to predict whether the areas of all the possible rectangular gardens will be the same or different. Students should record their predictions in their mathematics journals and then start on the investigation.

When they have finished, call the class together, and invite the students to share their findings. Work with an overhead projector and a transparency of the blackline master or draw a replica of the chart on the board. Fill in the chart with values offered by the students. Leave the area column blank for now.

Ask, “What discoveries did you make about the relationship between the perimeter of a rectangle and its length and width?” Students’ answers might include comments like the following:

- “Add all the lengths and widths to find the perimeter.”
- “First add the length and the width and then add these two numbers for the perimeter.”
- “Double the length, double the width, and then add these two numbers for the perimeter.”

This question may prompt the students to generalize a method for finding the perimeter of any rectangle. If not, ask additional process questions:

- “Did you need to measure every side of the rectangle to find the perimeter? Why, or why not?”
- How does knowing the characteristics of a rectangle help you find its perimeter?” (You need to measure only one length and one width, since a rectangle has two pairs of equal sides.)
- “If someone gave you the length and the width of any rectangle and asked you to find the perimeter, could you do it? If so, how?

Once students have deduced the formula, help them write it algebraically in two forms: $P = 21 + 2w$ and $P = 2 (l + w)$. Help students recognize that these two expressions are equivalent by expanding them and ordering terms to show both as $P = l + l + w + w$.

Next, complete the area column of the chart and ask the students for their observations. Students should note that the areas are not all the same. In fact, some areas will be considerably larger than others. Encourage the student to extend their observations by considering how changes in the length and width of a rectangle change its area. Ask your students to look back at their predictions about the areas of different gardens. Were their predictions accurate? Why, or why not? Students should write their responses to this question in their journals.

Ask the students which rectangle they think would make the best garden for their school, and why. (Responses and reasons will vary. Students will probably choose the rectangle that makes the biggest garden and should be able to explain that this rectangle has a greater area than the others even though the perimeters of all the rectangles remain constant.)

Depending on the location of the school and the space available for a garden, students might decide that one of the other rectangles would be more desirable for a garden than the largest rectangle, even though it offers less garden space. The figure below shows a sample of students’ work on the activity:

Sample work by students on Changing Garden

1. Complete the chart below.

FENCE	LENGTH	WIDTH	PERIMETER	AREA
A.	8	7	30 Feet	50 Sq. Feet
B.	9	6	30 Feet	54 Sq. Feet
C.	10	5	30 Feet	50 Sq. Feet
D.	11	4	30 Feet	44 Sq. Feet
E.	12	3	30 Feet	36 Sq. Feet
F.	13	2	30 Feet	26 Sq. Feet
G.	14	1	30 Feet	14 Sq. Feet

2. *In the space below, use words, pictures and/or numbers to describe how you found the perimeter of any size rectangle.* *To find a rectangle's perimeter, you take all of the lengths and all of the widths and add them together to find your perimeter of your rectangle.*

Extend

Have the students work together as a class to measure the perimeter of rectangular surfaces around the room. This time, encourage them to use fractions in expressing the dimensions. Students should first estimate the perimeter of an object and then measure the length and width and use the formula to find the actual perimeter.

Assessment Ideas

Ask your students to write two story problems-one related to area and the other related to perimeter. Give the students one condition for their problems: Nowhere may they use the word *area* or the word *perimeter.* Collect your students' problems and use them to create a page of story problems for everyone to solve.

Such exercises in writing and solving story problems related to perimeter and area will allow your students to demonstrate their ability to apply a specific strategy or formula. More important, these activities will provide students with opportunities to identify the attribute that they need to measure and the steps that they need to take when they are presented with a specific problem.

WHERE TO GO NEXT IN INSTRUCTION

After exploring the real-life applications with a constant perimeter and a changing area, like the changing garden, students should also explore real-life applications with a constant area and a changing perimeter. Students should find all possible rectangular configurations of a given area with integer side lengths. They should calculate the perimeter of each rectangle and analyze the "look" of the rectangles that have the smallest and largest perimeters.

For example, students could find all the different ways to arrange 48 square tiles to make a rectangular kitchen floor. After finding the perimeter of each rectangle, students should explore the changes in perimeter as the dimensions change and discuss which rectangle might make the most useful kitchen floor. The table below shows the possibilities:

Possible dimensions and perimeters of a kitchen floor made with 48 square tiles

FLOOR	LENGTH (Feet)	WIDTH (Feet)	AREA (Square Feet)	PERIMETER (Feet)
A	48	1	48	98
B	24	2	48	52
C	16	3	48	38
D	12	4	48	32
E	8	6	48	28

Activities in grades 3-5 should address the development of the ideas underlying formulas for perimeter and area. The next two activities provide informal settings in which students can explore the area formula in the context of parallelograms and triangles.